Fonctions usuelles (partie 3)

I) Cosinus et sinus hyperbolique

- $$\forall x \in \mathbb{R},\ ch(x) = \frac{e^x + e^{-x}}{2}$$
  $$\forall x \in \mathbb{R},\ sh(x) = \frac{e^x - e^{-x}}{2}$$

- $\forall x \in \mathbb{R},\ ch(x) > 0$ et elle est paire
  $$\left.\begin{array}{l}\forall x \in \mathbb{R}^+,\ sh(x) \geq 0 \\ \forall x \in \mathbb{R}^-,\ sh(x) \leq 0\end{array}\right\} \text{ et elle est impaire}$$

- $$\forall x \in \mathbb{R},\ ch'(x) = sh(x)$$
  $$\forall x \in \mathbb{R},\ sh'(x) = ch(x)$$

- $$\lim_{x \to +\infty} ch(x) = +\infty$$
  $$\lim_{x \to -\infty} ch(x) = +\infty$$
  $$\lim_{x \to +\infty} sh(x) = +\infty$$
  $$\lim_{x \to -\infty} sh(x) = -\infty$$

- $$\lim_{x\to 0}\frac{sh(x)}{x} = sh'(0) = 1 \text{ d'où } sh(x) \sim x \text{ (en 0)}$$
  $$\lim_{x\to 0}\frac{ch(x)-1}{x} = 0$$
  $$\lim_{x\to 0}\frac{ch(x)-1}{x^2} = \frac{1}{2}$$
- $$\forall x \in \mathbb{R}, \quad -ch(x) < sh(x) < ch(x)$$

II) Tangente hyperbolique

- $$\forall x \in \mathbb{R}, \; th(x) = \frac{sh(x)}{ch(x)} \quad \text{th est impaire}$$
- $$\forall x \in \mathbb{R}, \; th'(x) = 1 - th^2(x) = \frac{1}{ch^2(x)}$$
- $$\lim_{x\to +\infty} th(x) = 1$$
  $$\lim_{x\to -\infty} th(x) = -1$$
- $$\lim_{x\to 0}\frac{th(x)}{x} = 1 \text{ d'où } th(x) \sim x \text{ (en 0)}$$
- $$\forall x \in \mathbb{R}, \quad -1 < th(x) < 1$$

III) Cotangente hyperbolique

- $$\forall x \in \mathbb{R}^*, \quad \coth(x) = \frac{ch(x)}{sh(x)}$$
  $$\forall x \in \mathbb{R}^*, \quad \coth(x) = \frac{1}{th(x)}$$

- $$\forall x \in \mathbb{R}^*, \quad \coth'(x) = 1 - \coth^2(x) = \frac{-1}{sh^2(x)}$$

IV) Argsh

- $$\forall x \in \mathbb{R}, \quad \mathrm{Arg}sh(x) = y \Leftrightarrow \begin{cases} y \in \mathbb{R} \\ sh(y) = x \end{cases}$$
  $$\forall x \in \mathbb{R}, \quad \mathrm{Arg}sh(x) = \ln(x + \sqrt{1 + x^2})$$

- $$\lim_{x \to +\infty} Argsh(x) = +\infty$$
  $$\lim_{x \to -\infty} Argsh(x) = -\infty$$

- $$\forall x \in \mathbb{R}, \quad sh(\mathrm{Arg}sh(x)) = x$$
  $$\forall x \in \mathbb{R}, \quad \mathrm{Arg}sh(sh(y)) = y$$
  $$\forall x \in \mathbb{R}, \quad ch(\mathrm{Arg}sh(x)) = \sqrt{1 + x^2}$$

- $$\forall x \in \mathbb{R},\ \text{Arg}sh'(x) = \frac{1}{\sqrt{1+x^2}}$$
  $$\forall x \in \mathbb{R},\ \text{Arg}sh'(u) = \frac{u'}{\sqrt{1+u^2}}$$

- $$\int \frac{1}{\sqrt{1+x^2}}dx = Argsh(x) + C = \ln(x+\sqrt{1+x^2}) + C$$

V) Argch

- $$\forall x \in \mathbb{R}^+,\ Argch(x) = y \Leftrightarrow \begin{cases} \mathrm{y} \in \mathbb{R}^+ \\ ch(y) = x \end{cases}$$
  $$\forall x \in [1,+\infty[,\ Argch(x) = \ln(x+\sqrt{x^2-1})$$

- $$Argch(1) = 0$$
  $$\lim_{x \to +\infty} Argch(x) = +\infty$$

- $$\forall x \in [1,+\infty[,\ ch(Argch(x)) = x$$
  $$\forall y \in \mathbb{R}^+,\ Argch(ch(y)) = y$$
  $$\forall y \in \mathbb{R}^-,\ Argch(ch(y)) = -y$$
  $$\forall x \in [1,+\infty[,\ sh(Argch(x)) = \sqrt{x^2-1}$$

- $$\forall x\in]1,+\infty[,\ Argch'(x)=\frac{1}{\sqrt{x^2-1}}$$
  $$\forall x\in]1,+\infty[,\ Argch'(u)=\frac{u'}{\sqrt{u^2-1}}$$

- $$\forall x\in]1,+\infty[,\ \int\frac{1}{\sqrt{x^2-1}}dx=Argch(x)+C=\ln(x+(\sqrt{x^2-1})+C$$
  $$\forall x\in]-\infty,-1[,\ \int\frac{1}{\sqrt{x^2-1}}dx=-Argch(-x)+C=-\ln(-x+(\sqrt{x^2-1})+C$$

VI) Argth

- $$\forall x\in]-1,1[,\ Argth(x)=y \Leftrightarrow \begin{cases} y\in\mathbb{R} \\ th(y)=x \end{cases}$$
  $$\forall x\in [1,+\infty[,\ Argch(x)=\frac{1}{2}\ln(\frac{1+x}{1-x})$$

- $$\forall x\in]-1,1[,\ th(Argth(x))=x$$
  $$\forall y\in\mathbb{R},\ Argth(th(y))=y$$

- $$Argth(0)=0$$
  $$\lim_{x\to 1^-} Argth(x)=+\infty$$
  $$\lim_{x\to(-1)^+} Argth(x)=-\infty$$

- $$\forall x \in]-1,1[,\ Argth'(x) = \frac{1}{1-x^2}$$
  $$\forall x \in]-1,1[,\ Argth'(u) = \frac{u'}{1-u^2}$$

- $$\forall x \in]-1,1[,\ \int \frac{1}{1-x^2}dx = Argth(x) + C = \frac{1}{2}\ln(\frac{1+x}{1-x}) + C$$
  $$\forall x \notin]-1,1[,\ \int \frac{1}{1-x^2}dx = Argth(x) + C = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$$